MTH 132 - Quiz 3

9 June 2014

Name: Solutions.

Justify all your work to receive full credit. No notes, books, calculators, phones, or any electronic devices are allowed on this quiz. Good luck!

1. (12 points) Consider the function

$$f(x) = x^4 - 4x^3$$

in the following three parts.

(a) Use the derivative f' to determine where f is increasing or decreasing. [Hint: You can factor f'.]

$f'(x) = 4x^3 - 12x^2 = \underbrace{4x^2}_{\text{always positive}}\underbrace{(x-3)}_{\substack{\text{positive if } x>3 \\ \text{negative if } x<3}}$

So $f'(x) > 0 \iff x > 3$

Increasing	on	$(3, \infty)$	(or $[3, \infty)$)
Decreasing	on	$(-\infty, 3)$	(or $(-\infty, 3]$)

(b) Find the critical points of f. For each critical point, determine whether it is a local minimum, local maximum, or neither.

$f'(x) = 0$ if $\boxed{x = 0,\ x = 3}$

f' is always defined.

f' changes from (-) to (+) at $x = 3 \Rightarrow$ local minimum

f' doesn't change sign at $x = 0 \Rightarrow$ neither

(c) Find the maximum and minimum values of f on the interval $[0, 4]$.

Endpoints: $f(0) = 0$

$f(4) = 4^4 - 4 \cdot 4^3 = 0$

Critical pts: $f(3) = 3^4 - 4 \cdot 3^3$

$= -3^3 = -27$

Max: 0

Min: -27

2. (8 points) A car enters a 12-mile long tunnel at 12 : 00, and exits at 12 : 10. If the speed limit inside the tunnel is 60 miles per hour, can you say definitively whether the car was moving faster than the speed limit at some time?

Yes, using the Mean Value Theorem.

We have (s = position, v = velocity)

$$\left(\frac{\Delta s}{\Delta t} =\right) \frac{s(12:10) - s(12:00)}{12:10 - 12:00} = \frac{12 \text{ mile}}{10 \text{ min}} = 1.2 \text{ mile/min.}$$

$v(t)$ for some t, by the MVT, since $v = s'$.

But the speed limit is 1 mile/min,

so the car had to be speeding at some time.